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SUPER STOLARSKY-3 MEAN LABELING OF TRIANGULAR SNAKE GRAPHS

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Abstract

Let $G = (V, E)$ be a graph with p vertices and q edges. Let $\mathbf{f} : V(G) \to \{1, 2, \cdots, p+q\}$ be an injective function. For a vertex labeling $\mathbf{f}$, the induced edge labeling $\mathbf{f}^*(e = uv)$ is defined by

$$\mathbf{f}^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rceil \quad \text{(or)} \quad \left\lfloor \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rfloor$$

Then $\mathbf{f}$ is called a Super Stolarsky-3 Mean labeling if $f(V(G)) \cup \{f(e)/e \in E(G)\} = \{1, 2, \cdots, p+q\}$. A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs.
In this paper, we investigate Super Stolarsky-3 Mean labeling of Triangular Snake Graphs.

Key Words : *Graphs, Super Stolarsky-3 mean labeling, Path, Triangular snake graph, Double triangular snake graph, Triple triangular snake graph and Four triangular snake graph.*
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1. Introduction

All graphs $G = (V, E)$ with p vertices and q edges are finite, simple and undirected. For a detailed survey of graph labeling we refer Gallian(2017) [1]. For all other standard terminologies and notations we follow Harary [2]. S. Somasundaram and R. Ponraj introduced the concept of " **Mean Labeling of Graphs**" in 2004 [3] and S. Somasundaram and S. S. Sandhya introduced the concept of " **Harmonic Mean Labeling of graphs**" in [4]. S. S. Sandhya, E. Ebin Raja Merly and S. Kavitha introduced a new type of Labeling called "**Stolarsky-3 Mean Labeling of Graphs**" in [5]. In this paper we prove that Double Triangular Snake, Triple Triangular Snake, Four Triangular Snake graphs are Super Stolarsky-3 Mean labeling of graphs.

The following definitions and Theorems are useful for our present investigation.

A walk in which all the vertices $u_1, u_2, \cdots, u_n$ are distinct is called a path. It is denoted by P_n. **A Triangular Snake** T_n is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is, every edge of a path is replaced by a triangle C_3. **Double Triangular Snake** $D(T_n)$ consists of two Triangular snakes that have a common path. **Triple Triangular Snake** $T(T_n)$ consists of three Triangular snakes that have a common path. **Four Triangular Snake** $F(T_n)$ consists of four Triangular snakes that have a common path.

Definition 1.1 : Let $G = (V, E)$ be a graph with p vertices and q edges. Let $\mathbf{f}$: $V(G) \to \{1, 2, \cdots, p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^*(e = uv)$ is defined by

$$f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rceil \quad \text{(or)} \quad \left\lfloor \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rfloor$$

Then f is called a Super Stolarsky-3 Mean labeling if $f(V(G)) \cup \{f(e)/e \in E(G)\} = \{1, 2, \cdots, p+q\}$. A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs.

Theorem 1.2 [6] : Triangular Snake graph (T_n) is Super Stolarsky-3 Mean graph.

2. Main Results

Theorem 2.1 : Double Triangular Snake $D(T_n)$ is Super Stolarsky-3 Mean graph.

Proof : Consider a path $u_1, u_2, \cdots, u_n$.

Join u_i u_{i+1} to two new vertices v_i and w_i $1 \leq i \leq n-1$.

Define a function $f : V(D(T_n)) \rightarrow \{1, 2, \cdots, p+q\}$ by

$$\begin{aligned} \mathbf{f}(u_i) &= 8i-7, \quad 1 \leq i \leq n \\ \mathbf{f}(v_i) &= 8i-4, \quad 1 \leq i \leq n-1 \\ \mathbf{f}(w_i) &= 8i-2, \quad 1 \leq i \leq n-1 \end{aligned}$$

Then the edges are labeled with

$$\begin{aligned} \mathbf{f}(u_i\ u_{i+1}) &= 8i-3, 1 \leq i \leq n-1 \\ \mathbf{f}(u_i\ v_i) &= 8i-6, 1 \leq i \leq n-1 \\ \mathbf{f}(u_i\ w_i) &= 8i-5, 1 \leq i \leq n-1 \\ \mathbf{f}(v_i\ u_{i+1}) &= 8i-1, 1 \leq i \leq n-1 \\ \mathbf{f}(w_i\ u_{i+1}) &= 8i, 1 \leq i \leq n-1 \end{aligned}$$

Then the edge labels are distinct.
Hence $D(T_n)$ is Super Stolarsky-3 Mean graph.
Example 2.2 : The Super Stolarsky-3 Mean labeling of $D(T_4)$ is given below.

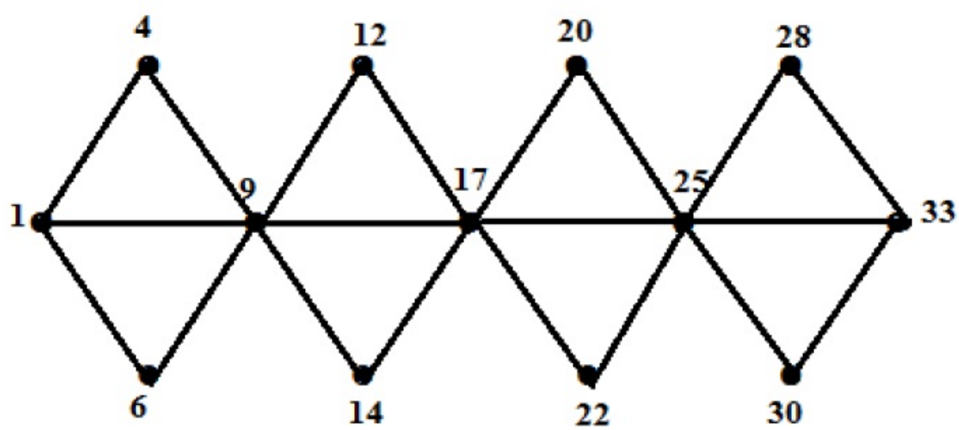

Figure:1

Theorem 2.3 : Triple Triangular Snake $T(T_n)$ is Super Stolarsky-3 Mean graph.
Proof : Let P_n be a path $u_1, u_2, \cdots, u_n$.
Join $u_i\ u_{i+1}$ to three new vertices v_i, w_i and x_i $1 \leq i \leq n-1$.

Define a function $\mathbf{f} : V(T(T_n)) \rightarrow \{1, 2, \cdots, p+q\}$ by

$$\begin{aligned}
\mathbf{f}(u_i) &= 11i - 10, \quad 1 \leq i \leq n \\
\mathbf{f}(v_i) &= 11i - 7, \quad 1 \leq i \leq n-1 \\
\mathbf{f}(w_i) &= 11i - 5, \quad 1 \leq i \leq n-1 \\
\mathbf{f}(x_i) &= 11i - 3, \quad 1 \leq i \leq n-1
\end{aligned}$$

Then the edges are labeled with

$$\begin{aligned}
\mathbf{f}(u_i\ u_{i+1}) &= 11i - 4, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ v_i) &= 11i - 9, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ w_i) &= 11i - 8, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ x_i) &= 11i - 6, 1 \leq i \leq n-1 \\
\mathbf{f}(v_i\ u_{i+1}) &= 11i - 2, 1 \leq i \leq n-1 \\
\mathbf{f}(w_i\ u_{i+1}) &= 11i - 1, 1 \leq i \leq n-1 \\
\mathbf{f}(x_i\ u_{i+1}) &= 11i, 1 \leq i \leq n-1.
\end{aligned}$$

Then the edge labels are distinct.

Hence $T(T_n)$ is Stolarsky-3 Mean graph.

Example 2.4 : The Super Stolarsky-3 Mean labeling of $T(T_4)$ is given below.

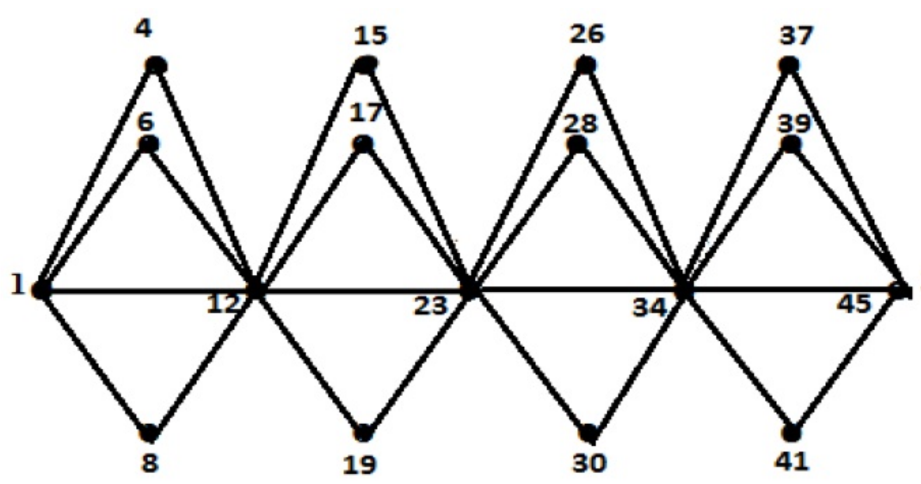

Figure: 2

Theorem 2.5 : Four Triangular Snake $F(T_n)$ is Super Stolarsky-3 Mean graph.

Proof : Let P_n be a path $u_1, u_2, \cdots, u_n$.

Join $u_i\ u_{i+1}$ to four new vertices v_i, w_i, x_i and y_i $1 \leq i \leq n-1$.

Define a function $\mathbf{f} : V(F(T_n)) \to \{1, 2, \cdots, p+q\}$ by

$$\begin{aligned}
\mathbf{f}(u_i) &= 14i - 13, \quad 1 \leq i \leq n \\
\mathbf{f}(v_i) &= 14i - 10, \quad 1 \leq i \leq n-1 \\
\mathbf{f}(w_i) &= 14i - 9, \quad 1 \leq i \leq n-1 \\
\mathbf{f}(x_i) &= 14i - 5, \quad 1 \leq i \leq n-1 \\
\mathbf{f}(y_i) &= 14i - 1, \quad 1 \leq i \leq n-1
\end{aligned}$$

Then the edges are labeled with

$$\begin{aligned}
\mathbf{f}(u_i\ u_{i+1}) &= 14i - 6, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ v_i) &= 14i - 12, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ w_i) &= 14i - 11, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ x_i) &= 14i - 8, 1 \leq i \leq n-1 \\
\mathbf{f}(u_i\ y_i) &= 14i - 7, 1 \leq i \leq n-1 \\
\mathbf{f}(v_i\ u_{i+1}) &= 14i - 4, 1 \leq i \leq n-1 \\
\mathbf{f}(w_i\ u_{i+1}) &= 14i - 3, 1 \leq i \leq n-1 \\
\mathbf{f}(x_i\ u_{i+1}) &= 14i - 2, 1 \leq i \leq n-1 \\
\mathbf{f}(y_i\ u_{i+1}) &= 14i, 1 \leq i \leq n-1.
\end{aligned}$$

Then the edge labels are distinct.

Hence $F(T_n)$ is Stolarsky-3 Mean graph.

Example 2.6 : The Super Stolarsky-3 Mean labeling of $F(T_4)$ is given below.

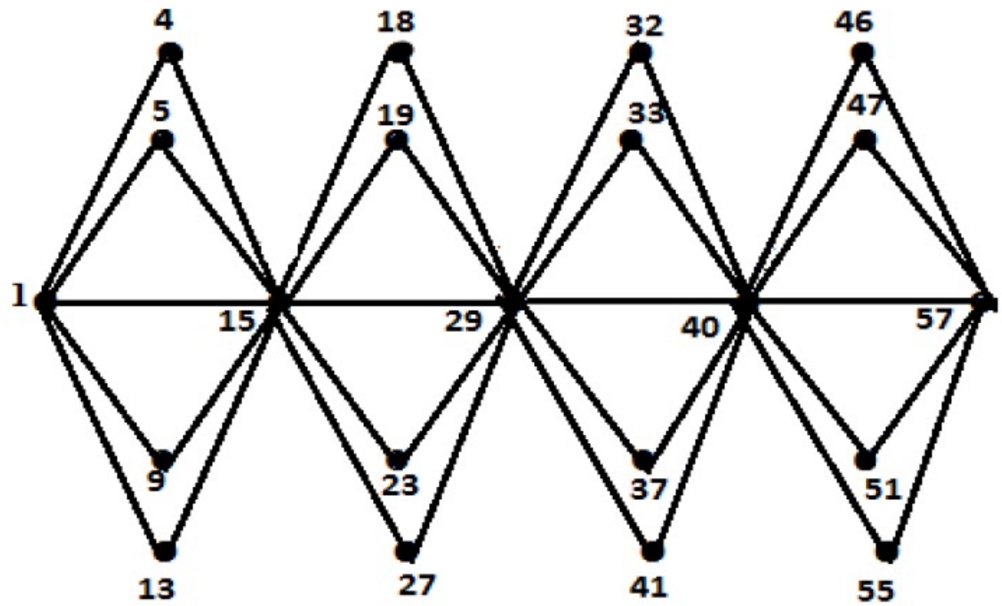

Figure: 3

3. Conclusion

In this paper, we discussed Super Stolarsky-3 Mean Labeling behavior of Double, Triple and Four Triangular Snake graphs. The authors are of the opinion that the study of Super Stolarsky-3 Mean labeling of Triangular Snake graphs shall be quite interesting and also will lead to newer results.

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